OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4730

Mechanics 3

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.
- You are permitted to use a graphic calculator in this paper.

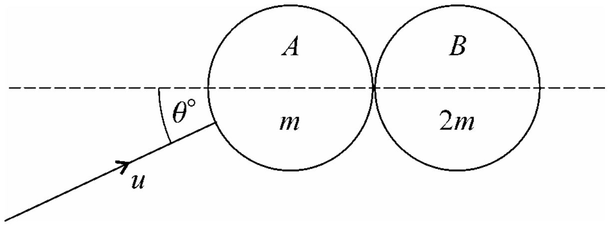
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 A particle is moving with simple harmonic motion in a straight line. The period is 0.2 s and the amplitude of the motion is 0.3 m. Find the maximum speed and the maximum acceleration of the particle. [6]

2



A sphere A of mass m, moving on a horizontal surface, collides with another sphere B of mass $2m$, which is at rest on the surface. The spheres are smooth and uniform, and have equal radius. Immediately before the collision, A has velocity u at an angle $\theta°$ to the line of centres of the spheres (see diagram). Immediately after the collision, the spheres move in directions that are perpendicular to each other.

(i) Find the coefficient of restitution between the spheres. [4]

(ii) Given that the spheres have equal speeds after the collision, find θ. [3]

3 An aircraft of mass 80 000 kg travelling at 90 m s^{-1} touches down on a straight horizontal runway. It is brought to rest by braking and resistive forces which together are modelled by a horizontal force of magnitude $(27\,000 + 50v^2)$ newtons, where v m s^{-1} is the speed of the aircraft. Find the distance travelled by the aircraft between touching down and coming to rest. [8]

4 For a bungee jump, a girl is joined to a fixed point O of a bridge by an elastic rope of natural length 25 m and modulus of elasticity 1320 N. The girl starts from rest at O and falls vertically. The lowest point reached by the girl is 60 m vertically below O. The girl is modelled as a particle, the rope is assumed to be light, and air resistance is neglected.

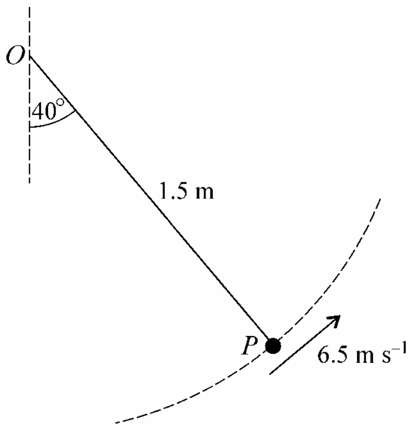
(i) Find the greatest tension in the rope during the girl's jump. [2]

(ii) Use energy considerations to find

(a) the mass of the girl, [4]

(b) the speed of the girl when she has fallen half way to the lowest point. [3]

5



A particle *P* of mass 0.3 kg is moving in a vertical circle. It is attached to the fixed point *O* at the centre of the circle by a light inextensible string of length 1.5 m. When the string makes an angle of $40°$ with the downward vertical, the speed of *P* is 6.5 m s^{-1} (see diagram). Air resistance may be neglected.

(i) Find the radial and transverse components of the acceleration of *P* at this instant. [2]

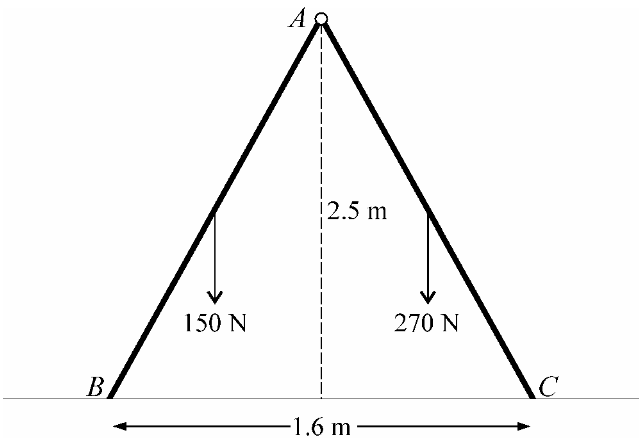
In the subsequent motion, with the string still taut and making an angle $\theta°$ with the downward vertical, the speed of *P* is $v \text{ m s}^{-1}$

(ii) Use conservation of energy to show that $v^2 \approx 19.7 + 29.4\cos\theta°$. [4]

(iii) Find the tension in the string in terms of θ. [4]

(iv) Find the value of *v* at the instant when the string becomes slack. [3]

6



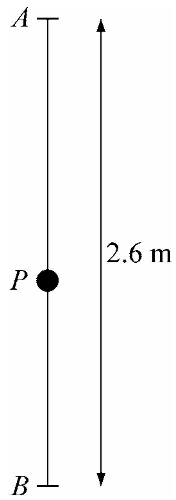
A step-ladder is modelled as two uniform rods *AB* and *AC*, freely jointed at *A*. The rods are in equilibrium in a vertical plane with *B* and *C* in contact with a rough horizontal surface. The rods have equal lengths; *AB* has weight 150 N and *AC* has weight 270 N. The point *A* is 2.5 m vertically above the surface, and $BC = 1.6$ m (see diagram).

(i) Find the horizontal and vertical components of the force acting on *AC* at *A*. [8]

(ii) The coefficient of friction has the same value μ at *B* and at *C*, and the step-ladder is on the point of slipping. Giving a reason, state whether the equilibrium is limiting at *B* or at *C*, and find μ. [6]

[Turn over

7



Two points A and B lie on a vertical line with A at a distance 2.6 m above B. A particle P of mass 10 kg is joined to A by an elastic string and to B by another elastic string (see diagram). Each string has natural length 0.8 m and modulus of elasticity 196 N. The strings are light and air resistance may be neglected.

(i) Verify that P is in equilibrium when P is vertically below A and the length of the string PA is 1.5 m. [4]

The particle is set in motion along the line AB with both strings remaining taut. The displacement of P below the equilibrium position is denoted by x metres.

(ii) Show that the tension in the string PA is $245(0.7+x)$ newtons, and the tension in the string PB is $245(0.3-x)$ newtons. [3]

(iii) Show that the motion of P is simple harmonic. [3]

(iv) Given that the amplitude of the motion is 0.25 m, find the proportion of time for which P is above the mid-point of AB. [5]

1 $0.2 = \frac{2\pi}{\omega} \Rightarrow \omega = 10\pi$	M1	For relevant use of $\frac{2\pi}{\omega}$
	A1	For correct value 10π
Hence maximum speed is $0.3 \times 10\pi = 3\pi \approx 9.42$ m s^{-1}	M1	For relevant use of $v = a\omega$
	A1√	For correct value 3π or 9.42
Maximum acc is $0.3 \times (10\pi)^2 = 30\pi^2 \approx 296$ m s^{-2}	M1	For relevant use of $a\omega^2$
	A1√ **6**	For correct value 30π or 296
	6	
3 **(i)** A and B move off ⊥ and || resp. to line of centres	M1	For correct directions of motion after impact
$2mv_B = mu\cos\theta$	A1	For correct momentum equation
$v_B = eu\cos\theta$	A1	For correct restitution equation
Hence $e = 0.5$	A1 **4**	For correct answer 0.5
(ii) $v_A = u\sin\theta$	B1	For correct equation
Hence $v_A = v_B \Rightarrow u\sin\theta = 0.5u\cos\theta$	M1	For forming the relevant equation for θ
So $\theta = \tan^{-1} 0.5 \approx 26.6°$	A1 **3**	For correct value 26.6
	7	
3 $80\,000v\frac{\mathrm{d}v}{\mathrm{d}x} = -(27\,000 + 50v^2)$	M1	For using Newton II to form a DE
	A1	For correct equation including $v\frac{\mathrm{d}v}{\mathrm{d}x}$
Hence $x = -\int \frac{1600v}{540+v^2}\,\mathrm{d}v$	M1	For separation of variables
$= -800\ln(540+v^2) + k$	M1	For logarithmic form of integral
	A1√	For correct integration of (their) $\frac{av}{b+cv^2}$
$v = 90$ when $x = 0 \Rightarrow k = 800\ln 8640$	M1	For use of initial condition to find k
Hence when $v = 0$, $x = 800\ln 16$	M1	For evaluation of required distance
		(The previous two M marks can equivalently be earned by using definite integration)
So distance is 2220 m approximately	A1 **8**	For correct value 2220
	8	
4 **(i)** Greatest tension $= \frac{1320 \times 35}{25} = 1848$ N	M1	For use of $\frac{\lambda x}{l}$ at lowest point
	A1 **2**	For correct answer 1848
(ii) **(a)** $mg \times 60 = \frac{1320}{2\times 25}(60-25)^2$	M1	For use of correct EPE formula $\frac{\lambda x^2}{2l}$
	A1	For correct unsimplified expression for EPE
Hence the girl's mass is 55 kg	M1	For use of equation involving EPE and GPE
	A1 **4**	For correct answer 55
(b) $55g \times 30 = \frac{1}{2} \times 55v^2 + \frac{1320}{2\times 25} \times (30-25)^2$	M1	For energy equation with KE, GPE and EPE
	A1√	For equation with all terms correct
So $v^2 = 564$, hence speed is 23.7 m s^{-1}	A1 **3**	For correct answer 24.3
	9	

5	(i)	Radial acc is $\frac{6.5^2}{1.5} = 28.2 \text{ m s}^{-2}$	B1		For correct value 28.2
		Transverse acc is $g\sin 40° = 6.30 \text{ m s}^{-2}$	B1	2	For correct value 6.30
	(ii)	$\frac{1}{2}\times 0.3\times(6.5^2 - v^2) = 0.3\times 9.8\times 1.5(\cos 40° - \cos\theta°)$	M1		For equating PE gain to KE loss, or equiv
			B1		For correct expression for PE gain
			B1		For correct expression for KE loss
		Hence $42.25 - v^2 = 29.4(\cos 40° - \cos\theta°)$			
		i.e. $v^2 \approx 19.7 + 29.4\cos\theta°$	A1	4	For showing given answer correctly
	(iii)	$T - 0.3g\cos\theta° = 0.3\times\frac{v^2}{1.5}$	M1		For use of Newton II, including $\frac{mv^2}{r}$ term
			A1		For correct (unsimplified) equation
		Hence $T = 2.94\cos\theta° + 0.2(19.7 + 29.4\cos\theta°)$	M1		For substitution, to obtain expression for T
		$= 3.95 + 8.82\cos\theta°$	A1	4	For correct answer
	(iv)	$T = 0$ when $3.95 + 8.82\cos\theta° = 0$	M1		For equating T to zero to find $\cos\theta$
		Hence $v^2 = 19.7 + 29.4\times\left(-\frac{3.95}{8.82}\right) \Rightarrow v \approx 2.56$	M1		For using this $\cos\theta$ to find v
			A1	3	For correct answer 2.56
				13	
6	(i)	Mom @ B for BAC: $V_C\times 1.6 = 150\times 0.4 + 270\times 1.2$	M1		For suitable moments equation for BAC
		Hence $V_C = 240$	A1		For correct value for V_C (or equivalent)
		Mom @ C for AC: $V_A\times 0.8 + H_A\times 2.5 = 270\times 0.4$	M1		For a moments equation for one rod with all required forces included
			A1		For a correct equation
		Res $\updownarrow$ for AC: $V_A + V_C = 270$	M1		For another equation leading to V_A
		Hence $V_A = 270 - 240 = 30$ N (upwards)	A1		For correct magnitude and direction
		and $2.5H_A = 108 - 0.8\times 30 \Rightarrow H_A = 33.6$ N (right)	M1		For substituting back to find H_A
			A1	8	For correct magnitude and direction
	(ii)	$V_B = 270 + 150 - V_C = 180$	M1		For finding all of V_B, H_B and H_C
			A1√		For correct V_B
		$H_B = H_C = H_A = 33.6$	A1√		For both H_B and H_C correct
		$\frac{H_B}{V_B} = \frac{33.6}{180} = 0.187$, $\frac{H_C}{V_C} = \frac{33.6}{240} = 0.14$	M1		For considering ratios at B and C, or equiv
		Hence friction is limiting at B	A1√		For identifying point with larger ratio
		Value of μ is 0.187	A1√	6	For identifying the larger ratio as μ
				14	

7	(i)	$T_{AP} = \frac{196}{0.8} \times (1.5 - 0.8) = 171.5$	M1		For using Hook's law to find either tension
		$T_{BP} = \frac{196}{0.8} \times (2.6 - 1.5 - 0.8) = 73.5$	A1		For both tensions correct
		$T_{AP} - T_{BP} = 98 = 10g$, hence equilibrium	M1		For considering $T_{AP} = mg + T_{BP}$, or equiv
			A1	4	For showing given result correctly
	(ii)	Extension of *PA* is $1.5 + x - 0.8 = 0.7 + x$	M1		For finding either extension in terms of x
		Hence $T_{AP} = \frac{196}{0.8}(0.7 + x) = 245(0.7 + x)$	A1		For showing one given answer correctly
		and $T_{BP} = \frac{196}{0.8}(1.1 - x - 0.8) = 245(0.3 - x)$	A1	3	For showing the other given answer correctly
	(iii)	$245(0.3 - x) + 10g - 245(07. + x) = 10\ddot{x}$	M1		For use of Newton II, at a general position
			A1		For a correct equation
		Hence $\ddot{x} = -49x$, so the motion is SHM	A1	3	For showing the given result correctly
	(iv)	$0.2 = 0.25\cos(7t)$	M1		For use of ± 0.2 in SHM equation involving t
			A1		For a correct equation for a relevant time
		Hence half of time above mid-pt is $t = 0.0919...$	A1		For correct value for a relevant time
		Proportion is $\frac{t}{\pi / \omega} = 0.205$	M1		For relating t to period of oscillation
			A1	5	For correct proportion 0.205
				15	